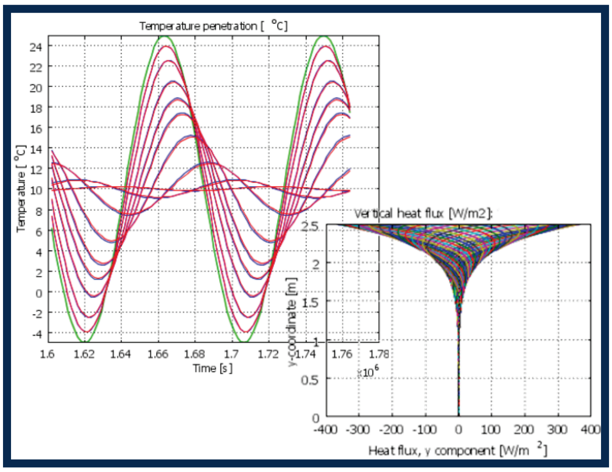
Impact of thermal diffusivity and thermal effusivity



1 Research goal

The thermal diffusivity and effusivity are both composite material properties that contain the qualities: density, conductivity and specific heat capacity.

The thermal effusivity is a measure of a materials ability to exchange thermal energy with its surroundings.

The thermal diffusivity of a material is a measure of how fast the material temperature adapts to the surrounding temperature. Generally they do not require much energy from their surroundings to reach thermal equilibrium.

The goal of this study is to visualize the impact of thermal diffusivity and effusivity separately in varying thermal surroundings.

3 Method

To reach the goal we used two materials that have the same thermal diffusivity but do not share the same thermal effusivity by far. The materials Rockwool and Sandstone meet these criteria.
We also used two materials for which the thermal effusivity is the same, but differ a lot in their values of thermal diffusivity. These materials are Asphalt and Gypsum.
As we mentioned before, the thermal diffusivity and effusivity can be calculated with the following material properties:

- thermal diffusivity $a = \frac{\lambda}{\rho c}$ $[m^2/s]$
- thermal effusivity $b = \sqrt{\lambda \rho c}$ $[W.s^{1/2}/m^2.K]$

With:

λ	=	thermal conductivity	[W/m.K]
ρ	=	density	$[kg/m^3]$
c	=	specific heat capacity	[J/kg.K]

The values for each material are the given in the table below:

Table 1 Thermal effusivity and diffusivity of the materials

	Thermal effusivity	Thermal diffusivity
Rockwool	22	**3*10^-6**
Sandstone	3005	**3*10^-6**
Asphalt	**785**	6,5*10^-8
Gypsum	**785**	1*10^-6

With these materials we want to visualize the impact of thermal effusivity and diffusivity on the temperature distribution and heat flux in the material by varying thermal environment.
We will simulate the material behavior by daily cyclic surface temperature variations with the software program COMSOL Multiphysics v3.5.

The results out of these simulations should give us information about thermal penetration, phase delay and energy fluxes in the materials. Some of these results will be reproduced by hand calculations.

4 The model

The physical module in Comsol that are applied to solve the heat problems is 'Heat Transfer by Conduction (ht2)'. The simulation model consist of four rectangles which each propose another material. The geometry and properties are mentioned below.

Geometry and mesh properties

<u>Dimensions [m]:</u>

Rectangles width = 0,2

Rectangles height = 2,5

In Fig. 1 the four 2D-models that are studied were shown. The colours in de left figure indicate the different material densities. In de right figure the colours indicate the thermal conductivity of the four materials.

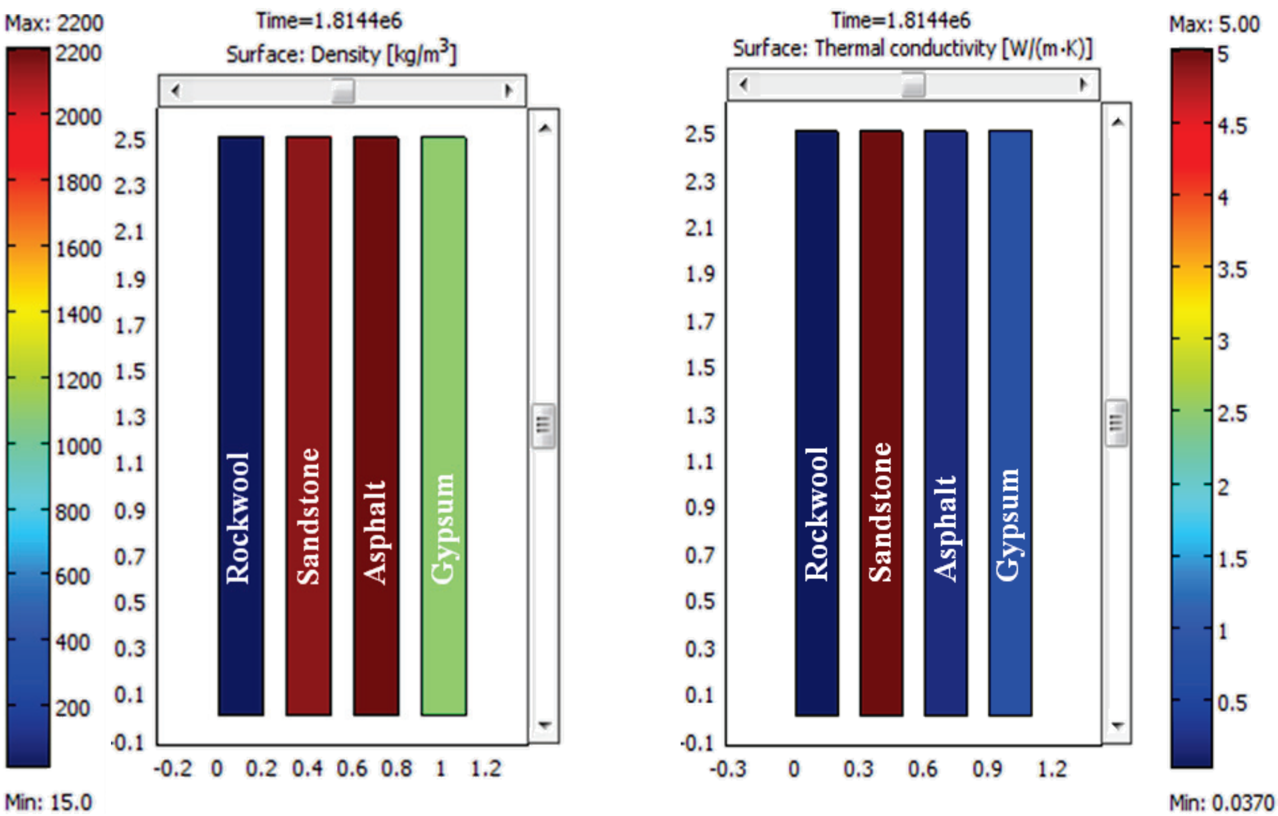


Fig. 1 Geometry of the simulation models, left: the colors indicate material density; right: colors indicate the thermal conductivity.

Mesh settings:

The mesh of the simulation model consists of 4.704 elements.

Below (Fig. 2) the mesh geometry of the model are given. On the right side the accompanying mesh statistics are presented:

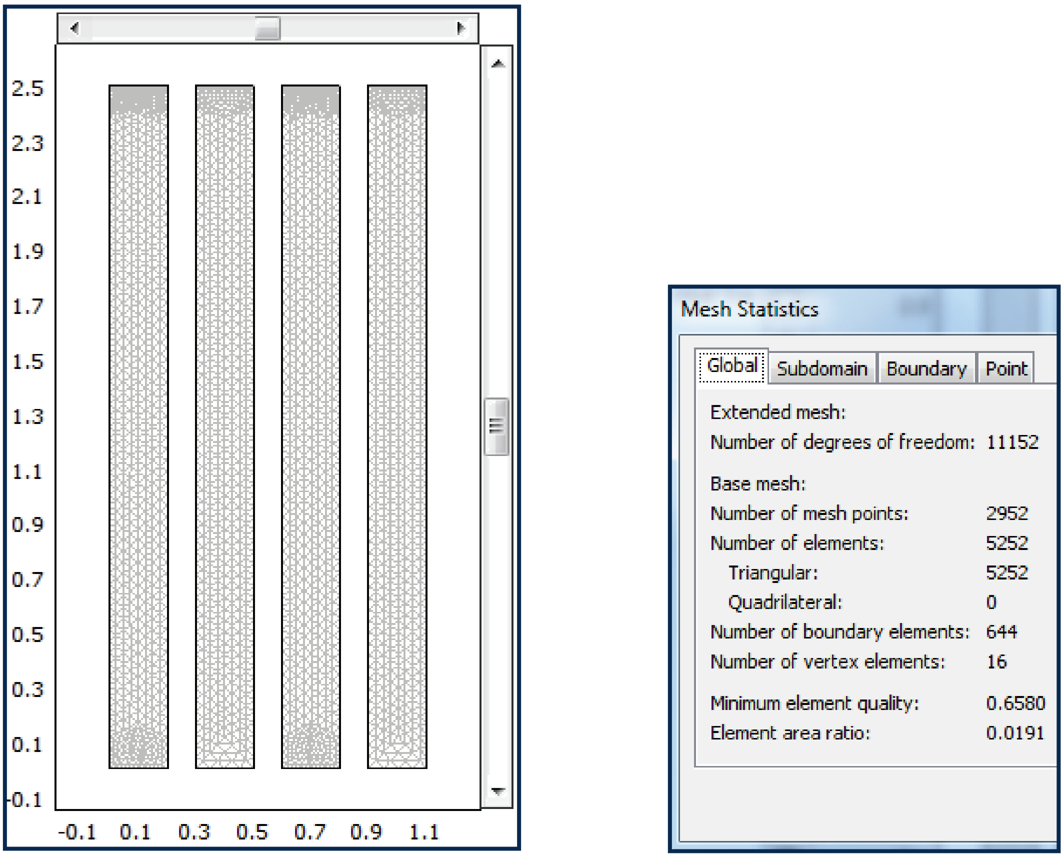


Fig. 2 Mesh geometry (lowest half meter) of the simulation mode (lowest half meter), and mesh statistics (right)

Boundary Conditions

Temperature:

Upper boundary temperature = $283.15 + 15 \cdot \sin(\frac{2\pi \cdot t}{24 \cdot 3600})$ K

The surface temperature is a simple sine curve with an oscillation period of 24 hours. The curve has an amplitude of 15 degrees Celsius around a base temperature of 10 degrees Celsius.

Thermal insulation boundaries = bottom, left en right boundary (all others)

Material Properties and Initial Condition

Thermal conductivity k:

Rockwool	= 0,037 [W/m.K]
Sandstone	= 5,000 [W/m.K]
Asphalt	= 0,200 [W/m.K]
Gypsum	= 0,800 [W/m.K]

Density ρ:

Rockwool	= 15 [kg/m^3]
Sandstone	= 2150 [kg/m^3]
Asphalt	= 2200 [kg/m^3]
Gypsum	= 1100 [kg/m^3]

Specific heat capacity c:

Rockwool	= 840 [J/kg.K]
Sandstone	= 840 [J/kg.K]
Asphalt	= 1400 [J/kg.K]
Gypsum	= 700 [J/kg.K]

Initial temperature condition = 283,15 [K]

Solver Settings

Type of analysis: Transient Heat Transfer by Conduction (ht2)

Linear system solver: Direct (UMFPACK)

Relative tolerance: 0,0001

Absolute tolerance: 0,00001

Pivot threshold: 0,1

Memory allocation factor: 0,7

Simulation time: 3 weeks, range(0,1800,21*24*3600)

5 Results

In this section both simulation as calculation results were presented.

Simulation results

Below, the material temperatures are given at a specific time. Spread over the daily period, after each 4 hours a figure shows the temperature of the materials at that time. Note that the temperature range is different in each figure:

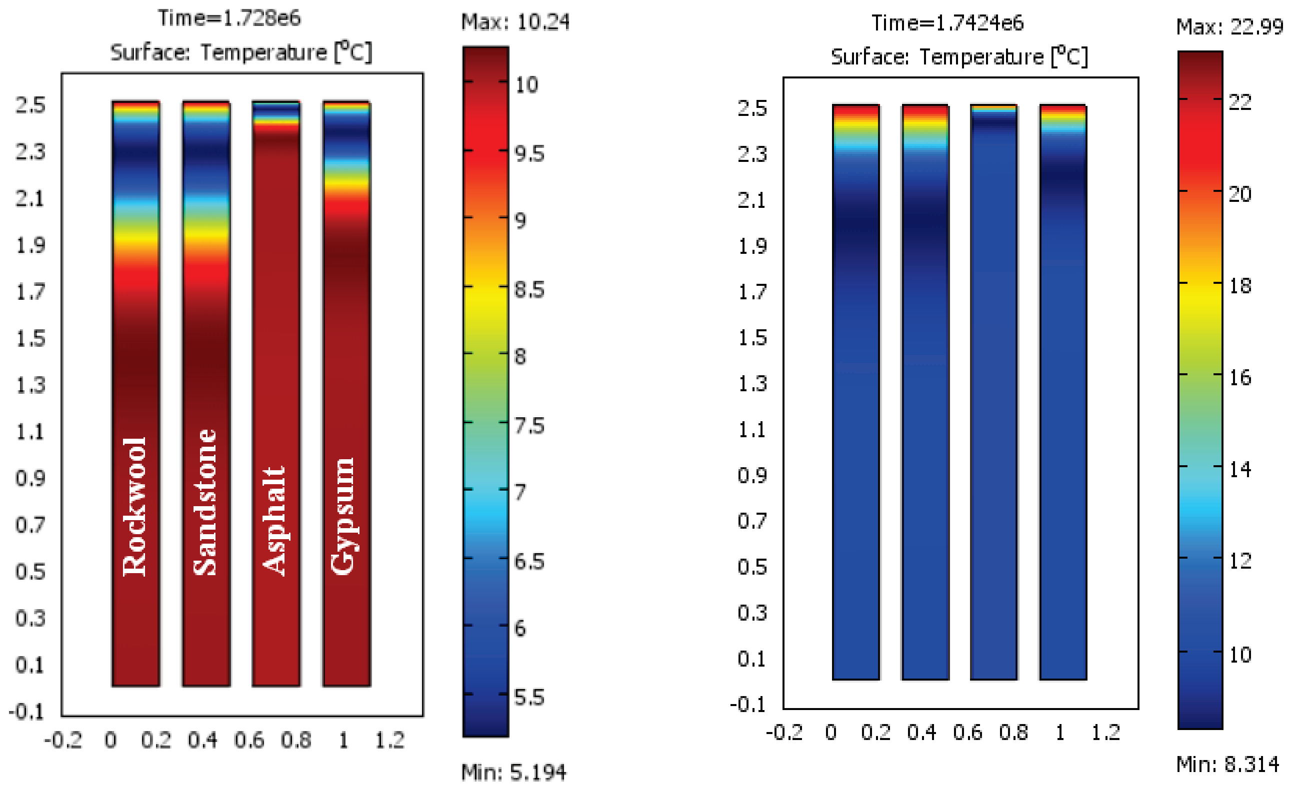


Fig. 3 Material temperatures; left: at time = 0 hours, right: at time = 4 hours

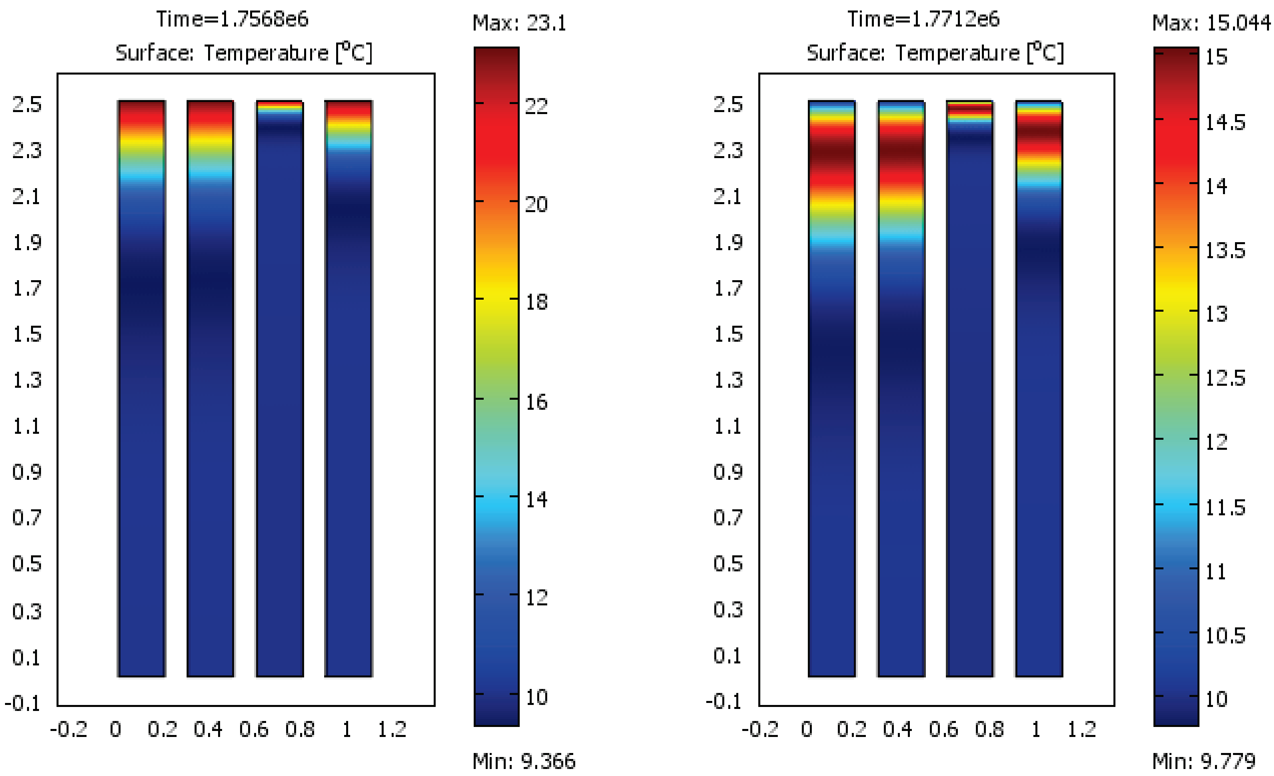


Fig. 4 Material temperatures; left: at time = 8 hours, right: at time = 12 hours

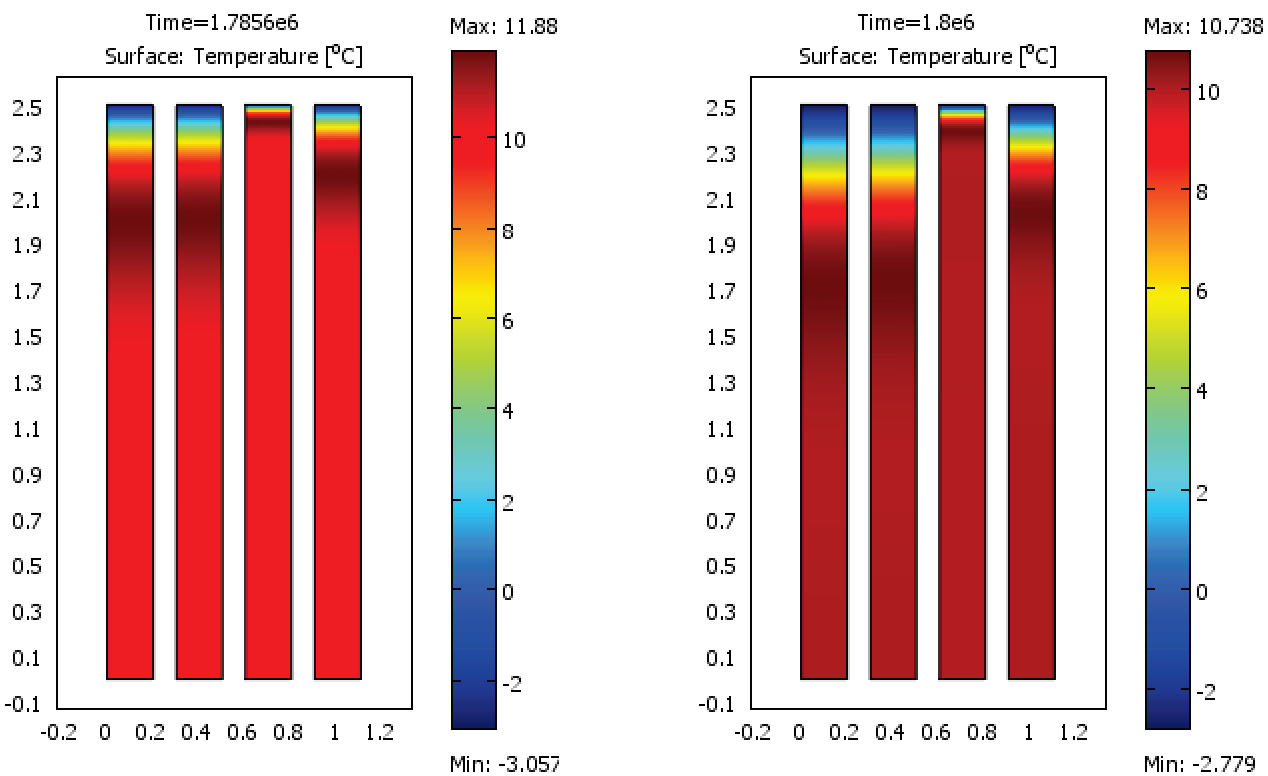
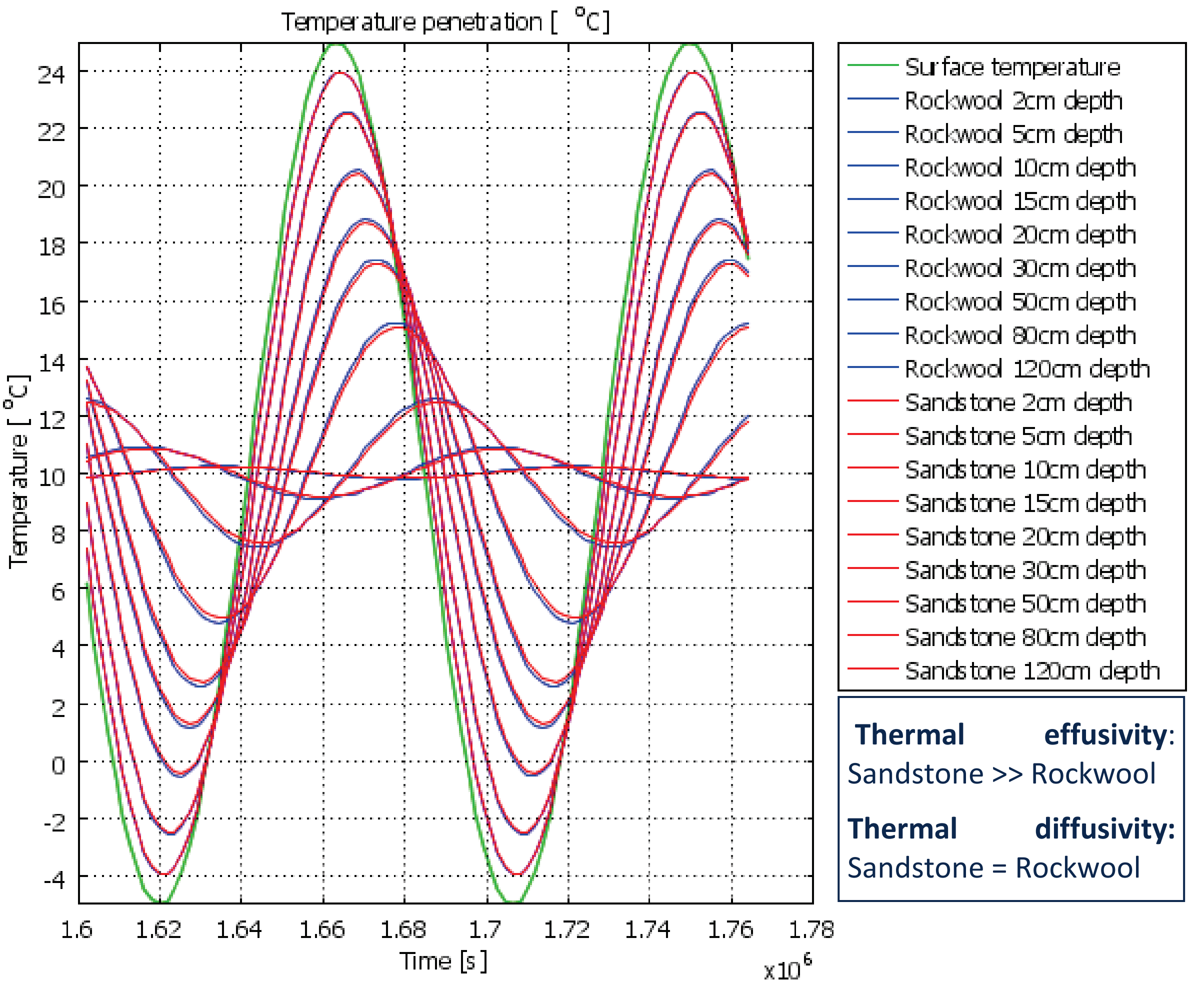


Fig. 5 Material temperatures; left: at time = 16 hours, right: at time = 20 hours

In the graphs below the penetration of the cyclic surface temperature in the materials is more clearly visualized at several depths during almost two days.

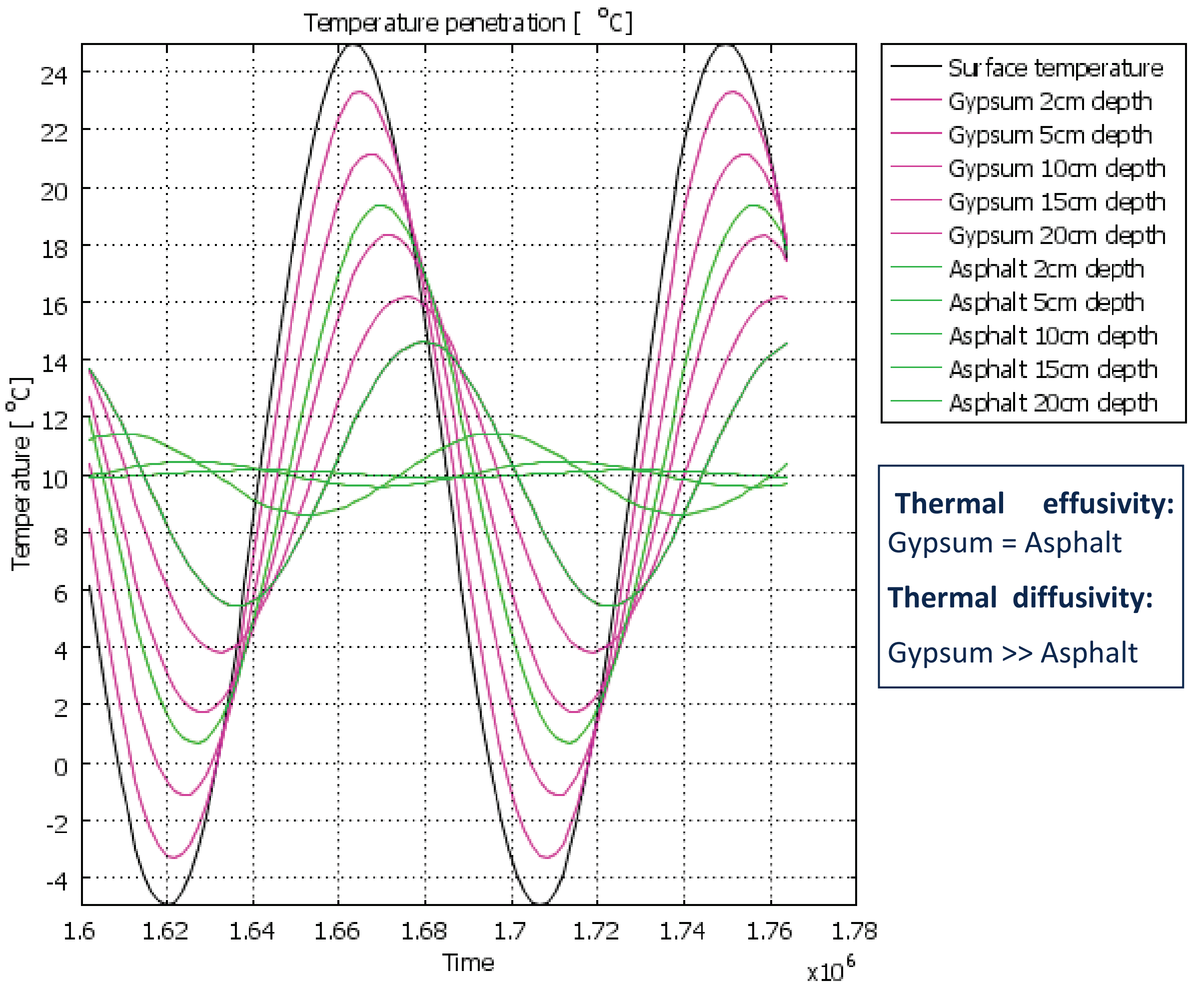
In Graph 1 the materials Rockwool (blue) and Sandstone (red) are compared with each other:

Graph 1 Temperature penetration in Rockwool and Sandstone



In Graph 2 the temperature penetration in the materials Gypsum (violet) and Asphalt (green) are compared with each other:

Graph 2 Temperature penetration in Gypsum and Asphalt

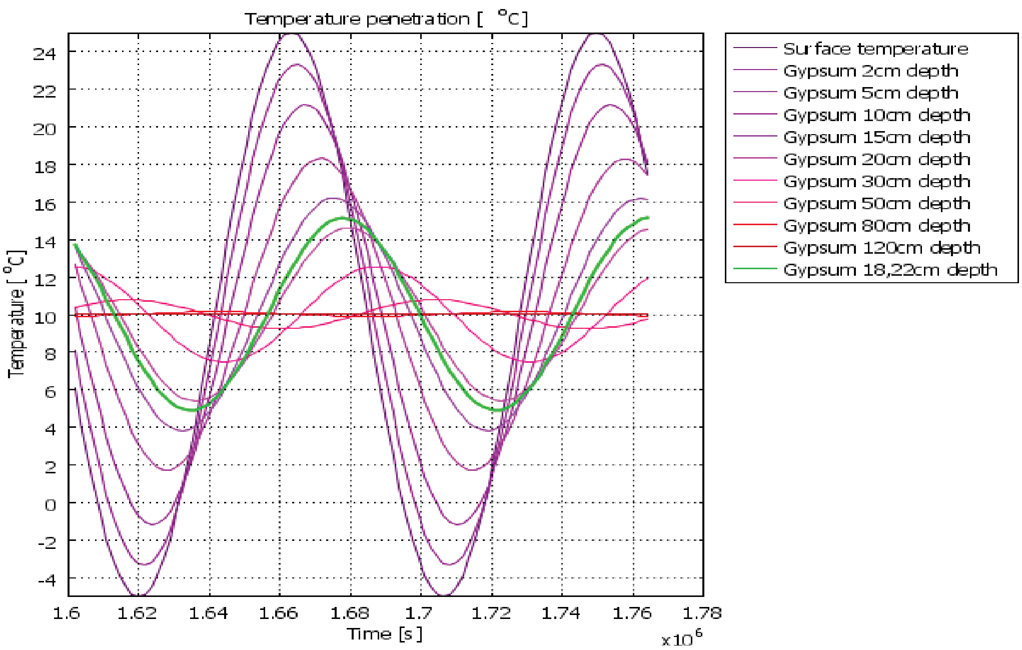


Thermal effusivity:
Gypsum = Asphalt

Thermal diffusivity:
Gypsum >> Asphalt

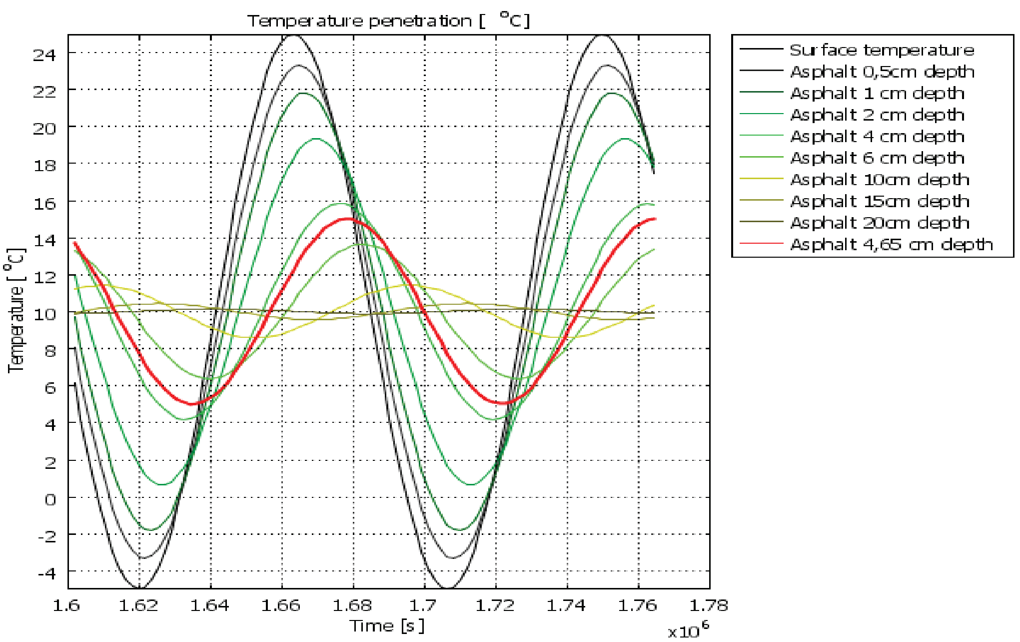
Because the temperature penetration in Gypsum and Asphalt are quite different, we separate these results in two graphs with custom depths. In Graph 3 the temperature penetration in Gypsum is presented at different depths. The green line shows the temperature at 18,22 cm depth. According to the hand calculations the amplitude at this depth should have been reduced to 33,3% (= 5°) of the surface temperature amplitude (15°C).

Graph 3 Temperature penetration in Gypsum



In Graph 3 the temperature penetration in Asphalt is presented at different depths. The red line shows the temperature at 4,65 cm depth. According to the hand calculations the amplitude at this depth should been reduced to 33,3% (= 5°) as well.

Graph 4 Temperature penetration in Asphalt



The graphs below visualize the penetration of the cyclic surface temperature and vertical heat fluxes through the whole material at several times during a day.

In Rockwool:

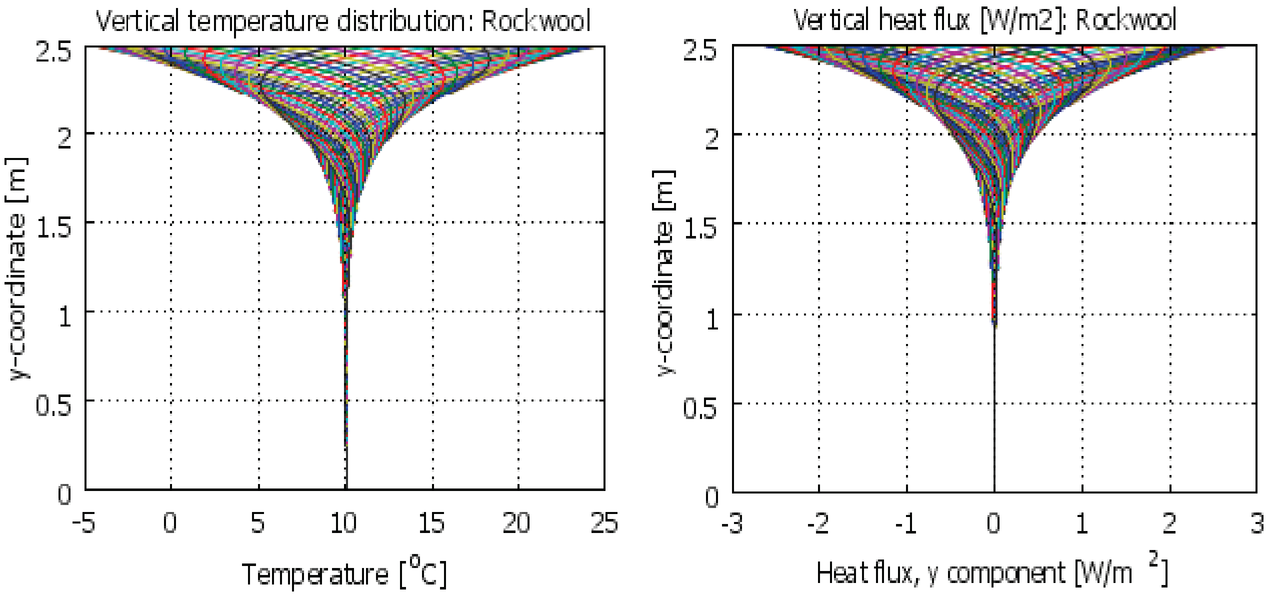


Fig. 6 Rockwool: temperature penetration left, and vertical heat flux on the right.

In Sandstone:

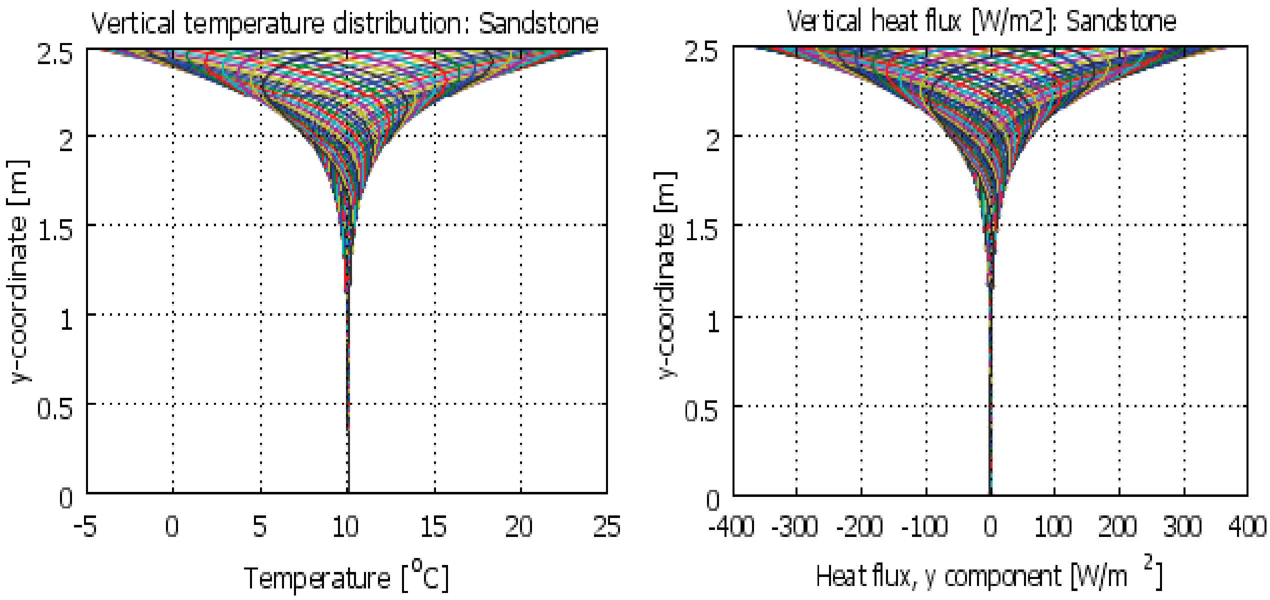


Fig. 7 Sandstone: temperature penetration left, and vertical heat flux on the right.

In Asphalt:

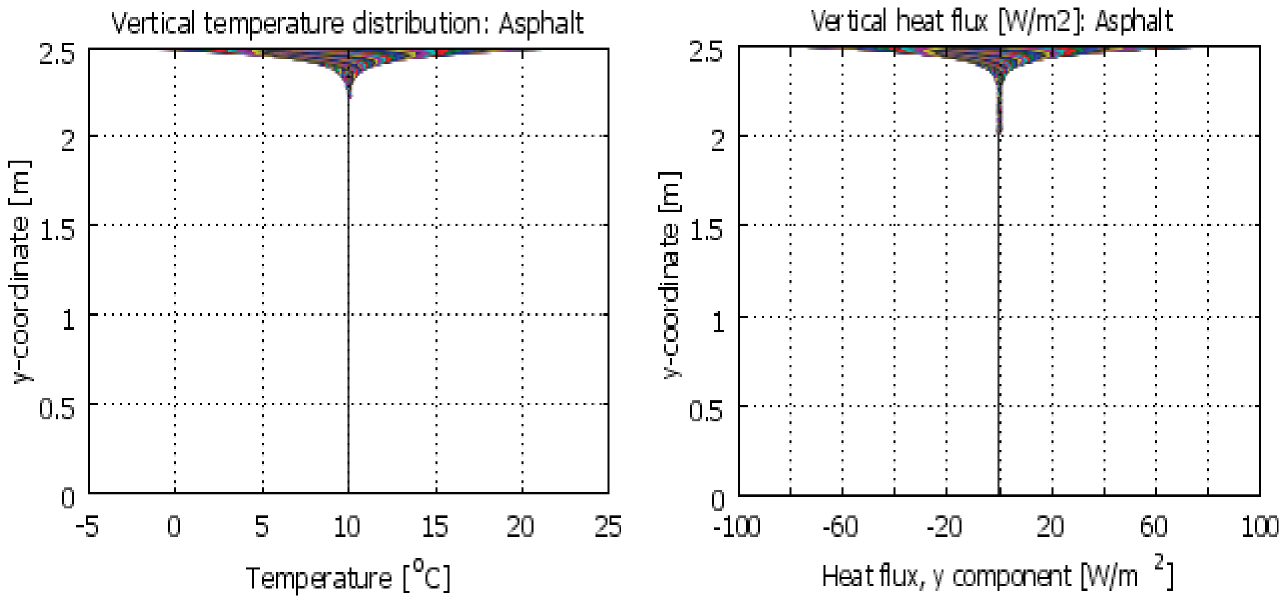


Fig. 8 Asphalt: temperature penetration left, and vertical heat flux on the right.

In Gypsum:

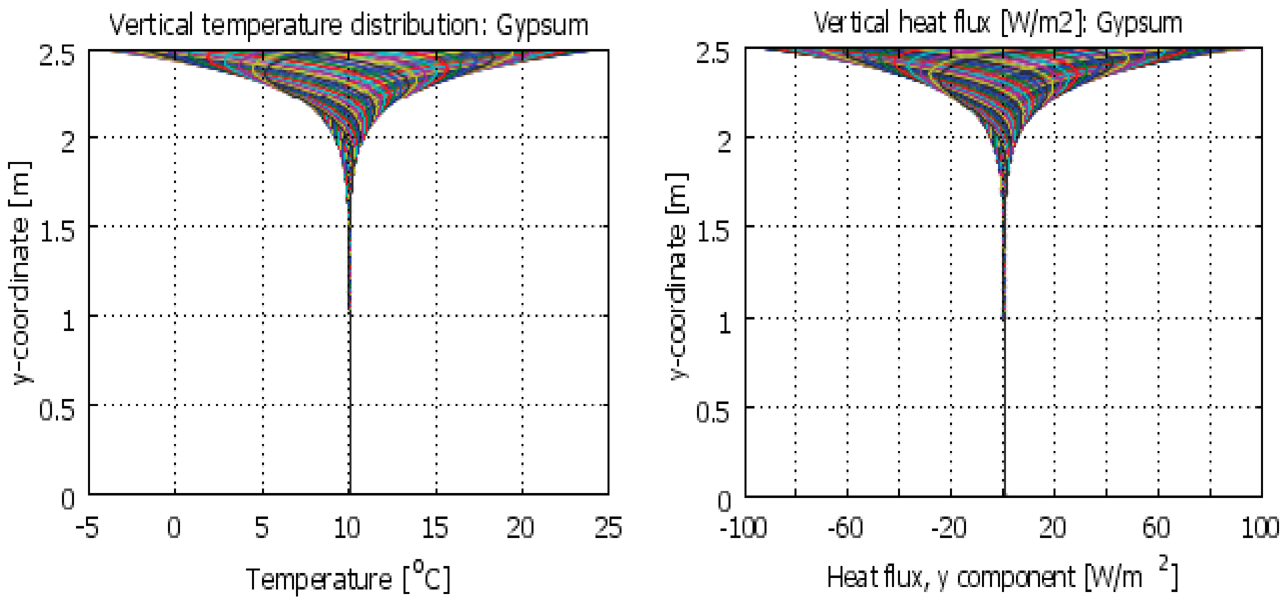
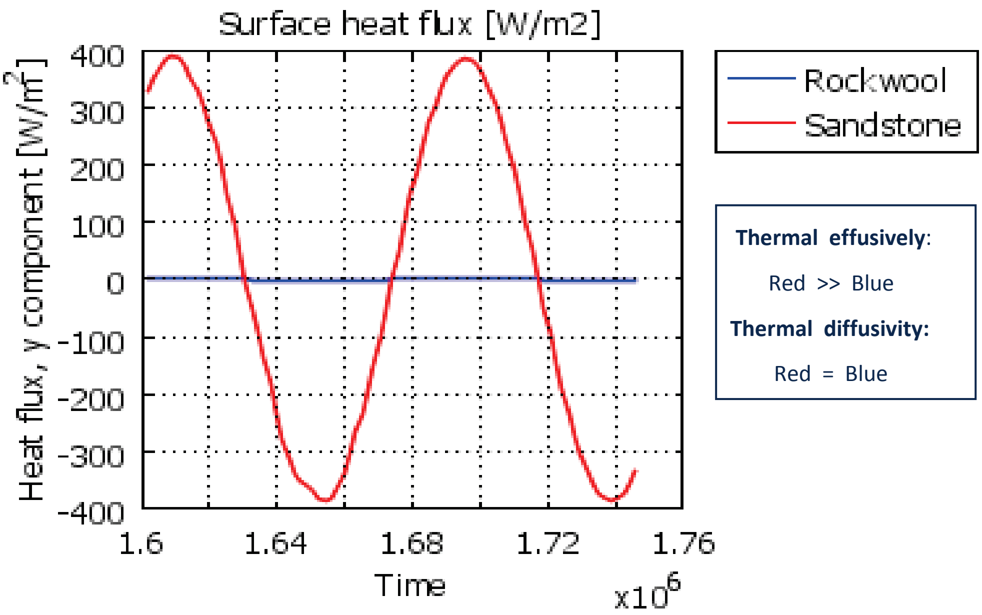


Fig. 9 Gypsum: temperature penetration left, and vertical heat flux on the right.

The following graphs show the varying surface energy flux due to the cyclic surface temperature during about two days.

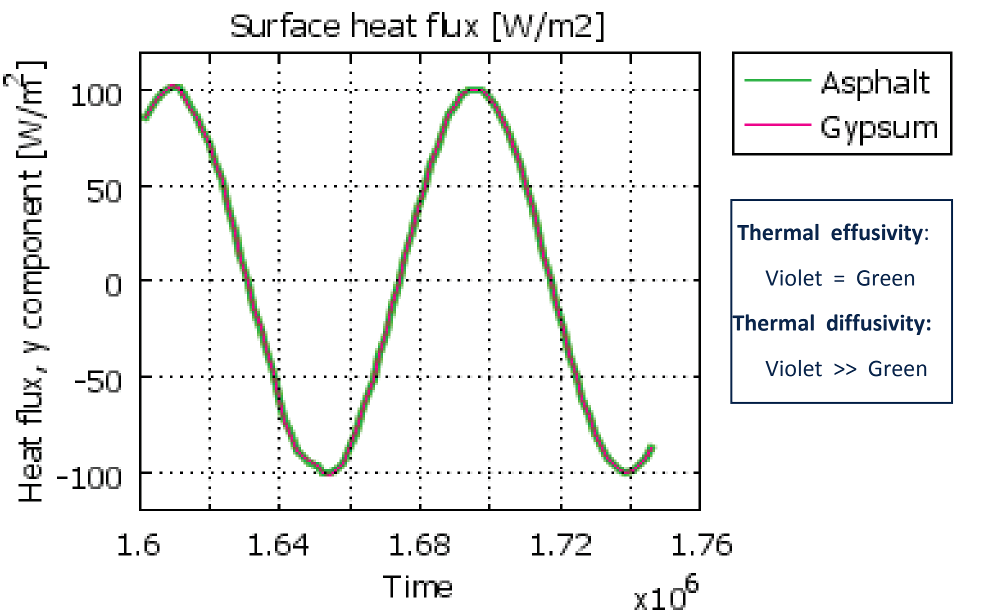
In Graph 5 the materials Rockwool (blue) and Sandstone (red) are compared with each other:

Graph 5 Heat flux at the surface of Rockwool and Sandstone



In Graph 6 the materials Asphalt (green) and Gypsum (violet) are compared with each other, the heat flux is the same for both materials.

Graph 6 Heat flux at the surface of Asphalt and Gypsum



During a cyclic surface temperature, the 'penetration depth' of a semi-infinite thick slab can be calculated as follows:

$$d^* = \sqrt{\frac{a t_0}{\pi}} \quad [m]$$

With:

d^*	=	penetration depth [m]
a	=	thermal diffusivity [m^2/s]
t_0	=	period [s]

The phase time shift of the cycles at a certain dept x is calculated by:

$$\Delta t = \frac{x}{2\pi d^*} t_0 \quad [s]$$

With:

Δt	=	time shift [s]
x	=	depth in material [m]

The temperature of a semi-infinite thick slab at a certain dept x, and a certain time t, can be calculated as follows:

$$T_{(x,t)} = \hat{T}_s \cdot e^{-x/d^*} \cdot \cos(\omega \cdot t - x/d^*) + T_0$$

With:

ω	=	Angular frequency [rad/s]
$\hat{T}_s$	=	Surface temperature amplitude [K]
T_0	=	Surface mean temperature [K]

So at a depth x = d*, the amplitude is only 1/e of the value on the surface. At depth x = 3d* the amplitude is only 5% of the value on the surface.

The varying heat flow at the surface of a semi-infinite thick wall is calculated by:

$$\widetilde{Q_s} = (1 + j) \cdot b \cdot \sqrt{\frac{\pi}{2 \cdot t_0}} \cdot \widetilde{T_s} \quad [W/m^2]$$

Were the peak load of the heat flow curve is:

$$\widehat{Q}_s = b \cdot \sqrt{\frac{\pi}{2 \cdot t_0}} \cdot \widehat{T}_s \quad [W/m^2]$$

With:

b = thermal effusivity [$W.s^{1/2} / m^2.K$]

$\widetilde{T}_s$ & $\widehat{T}_s$ = Surface temperature curve respectively temperature amplitude [K]

Using these formula's we have calculated the following values for each material:

	Thermal effusivity	Thermal diffusivity	Penetration dept d*	Depth were amplitude reduced to 33% (=5°C)	Depth were phase shift = 1 hour	Depth were phase shift = 4 hour	Depth were phase shift = 7 hour	Surface peak heat flow
Rockwool	22	**3*10^-6**	28,7 cm	**31,6 cm**	**7,5 cm**	**30,1 cm**	**52,6 cm**	22,8 w/m2
Sandstone	3005	**3*10^-6**	28,7 cm	**31,6 cm**	**7,5 cm**	**30,1 cm**	**52,6 cm**	384,4 w/m2
Asphalt	**785**	6,5*10^-8	4,2 cm	4,7 cm	1,1 cm	4,4 cm	7,8 cm	**100,4 w/m2**
Gypsum	**785**	1*10^-6	16,6 cm	18,2 cm	4,34 cm	17,4 cm	30,4 cm	**100,4 w/m2**

6 Conclusion

Both simulation and calculation results show us clearly that the thermal diffusivity of a material influences the penetration dept and speed of temperature adaption under a varying thermal environment. The thermal diffusivity says nothing about the energy flows.

On the other hand the thermal effusivity influences the ability to exchange thermal energy with its surroundings. In materials with a high thermal effusivity, the energy flux will also be high when there are temperature differences.

Because Rockwool and Sandstone have the same thermal diffusivity, the temperature distribution in the materials will be the same, as can be seen in Graph 1. However, the vertical heat flux is far from the same, as can be seen in Graph 5.

Because Gypsum and Asphalt have the same thermal effusivity and different diffusivities, these results are reversed, as can be seen in Graph 2 and Graph 6.

The results out of the hand calculations are in good agreement with the simulation results. As can be seen in Graph 3 and Graph 4, is the amplitude at the calculated depth indeed only 33,3% of the amplitude at the surface. Also the calculated surface peak heat flow is nearly the same as simulated.

Bibliography

M.h. de Wit, [2009]: *Heat, air and moisture in building envelopes.* Course book Eindhoven University of Technology: blz 63-69